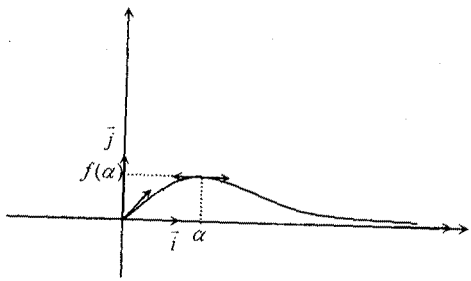
x	0		α		$+\infty$
$f'(x)$	0	$+$	0	$-$	
$f(x)$	0	↗	$\frac{2\alpha}{\alpha^2+1}$	↘	0

- $\lim\limits_{x\to+\infty} f(x) = \lim\limits_{x\to+\infty} \dfrac{\ln(x^2+1)}{x} = \lim\limits_{x\to+\infty} \dfrac{\ln x^2(1+\frac{1}{x^2})}{x}$

$= \lim\limits_{x\to+\infty} \dfrac{\ln x^2}{x} + \dfrac{\ln(1+\frac{1}{x^2})}{x}$ Or $\lim\limits_{x\to+\infty} \dfrac{\ln x^2}{x} = \lim\limits_{x\to+\infty} \dfrac{2\ln x}{x} = 0$ et $\lim\limits_{x\to+\infty} \dfrac{\ln(1+\frac{1}{x^2})}{x} = 0$ d'où $\lim\limits_{x\to+\infty} f(x) = 0$.

c)



Solution 6 :

1°/a) $D_h = \{ x \in \mathbf{R} / \dfrac{x+1}{x} > 0,\ x \neq 0 \text{ et } 2x+1 \neq 0 \}$.

$D_h =]-\infty,-1,[\cup]0,+\infty[$.

h est définie, continue et dérivable sur $]-\infty,-1,[\cup]0,+\infty[$

$\forall x \in D_h,\ h'(x) = \dfrac{\left(\frac{x+1}{x}\right)'}{\frac{x+1}{x}} + \dfrac{2}{(2x+1)^2} = \dfrac{-1}{x(x+1)} + \dfrac{2}{(2x+1)^2}$